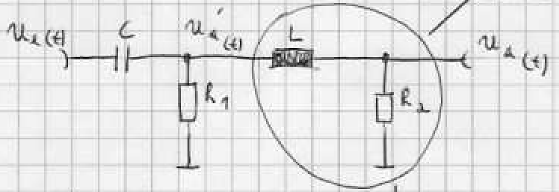
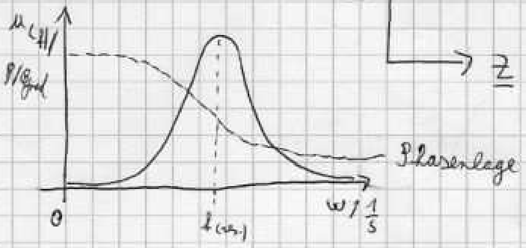
[Teil 1]

Übertragungsfunktion Bandpass – Herleitung!



$\underline{u_a} = \underline{u_a'} \cdot \dfrac{R_2}{R_2 + j\omega L}$

$\underline{Z} = R_2 + j\omega L$



$$H(j\omega) = \frac{\underline{u_a'}}{\underline{u_e}} = \frac{\dfrac{R_1 \cdot (R_2 + j\omega L)}{R_1 + R_2 + j\omega L}}{\dfrac{1}{j\omega C} + \dfrac{R_1 (R_2 + j\omega L)}{R_1 + R_2 + j\omega L}} \quad \begin{matrix} | \cdot j\omega C \cdot (R_1 + R_2 + j\omega L) \\ \\ | \cdot j\omega C \cdot (R_1 + R_2 + j\omega L) \end{matrix}$$

$$\underline{u_a'} = \underline{u_e} \cdot \frac{\dfrac{R_1 (R_2 + j\omega L) \cdot j\omega C \cdot \cancel{(R_1 + R_2 + j\omega L)}}{\cancel{(R_1 + R_2 + j\omega L)}}}{R_1 + R_2 + j\omega L + \dfrac{R_1 (R_2 + j\omega L) \cdot j\omega C \cancel{(R_1 + R_2 + j\omega L)}}{\cancel{(R_1 + R_2 + j\omega L)}}}$$

$$= \underline{u_e} \cdot \frac{j\omega R_1 C (R_2 + j\omega L)}{R_1 + R_2 + j\omega L + j\omega R_1 C (R_2 + j\omega L)}$$

$$= \underline{u_e} \cdot \frac{j\omega R_1 R_2 C - \omega^2 R_1 L C}{R_1 + R_2 + j\omega L + j\omega R_1 R_2 C - \omega^2 R_1 L C} \cdot \frac{R_2}{R_2 + j\omega L}$$

[Teil 2]

Übertragungsfunktion Bandpass

— Teil 2 —

$$\frac{\underline{u}_a}{\underline{u}_e} = \frac{j\omega R_1R_2^2C - \omega^2R_1R_2LC}{R_2(R_1+R_2) + j\omega R_2L + j\omega R_1R_2^2C - \omega^2R_1R_2LC + j\omega(R_1+R_2)L - \omega^2L^2 - \omega^2R_1R_2LC - j\omega^3R_1L^2C}$$

$$= \frac{j\omega R_1R_2^2C - \omega^2R_1R_2LC}{\left[R_2(R_1+R_2) - 2\omega^2R_1R_2LC - \omega^2L^2\right] + j\left[\omega R_2L + \omega R_1R_2^2C + \omega(R_1+R_2)L - \omega^3R_1L^2C\right]}$$

$$= \frac{j\omega R_1R_2^2C - \omega^2R_1R_2LC}{\left[R_2(R_1+R_2) - \omega^2(L^2+2R_1R_2LC)\right] + j\left[\omega\left(R_1R_2^2C + \underbrace{(R_1+R_2)L + R_2L}_{\to\, R_1L + 2R_2L}\right) - \omega^3R_1L^2C\right]}$$

Gesamtausdruck – ~~Eulersche~~ Normal Form:

$$\frac{\underline{u}_a}{\underline{u}_e} = \frac{\omega^2R_1R_2LC - j\omega R_1R_2^2C}{\left[\omega^2(L^2+2R_1R_2LC) - R_2(R_1+R_2)\right] - j\left[\omega\left(R_1R_2^2C + (R_1+R_2)L + R_2L\right) - \omega^3R_1L^2C\right]}$$

Gesamtausdruck – Eulersche Form:

$$\frac{\underline{u}_a}{\underline{u}_e} = \frac{\sqrt{(\omega^2R_1R_2LC)^2 + (\omega R_1R_2^2C)^2}}{\sqrt{\left[\omega^2(L^2+2R_1R_2LC) - R_2(R_1+R_2)\right]^2 + \left[\omega\left(R_1R_2^2C + R_1L + 2R_2L\right) - \omega^3R_1L^2C\right]^2}} \cdot e^{j\varphi}$$

$\varphi \to$ siehe nächste Seite!

[Teil 3]

Übertragungsfunktion Bandpass

$$\| \varphi = \varphi_{(Zähler)} - \varphi_{(Nenner)} \|$$

$$\tan\varphi_1 = -\frac{\omega^2 \cancel{R_1} R_2^{\cancel{2}} \cancel{C}}{\omega^2 \cancel{R_1} \cancel{R_2} L \cancel{C}} = -\frac{R_2}{\omega L} \; ; \quad \tan\varphi_2 = -\frac{\omega(R_1R_2^2C + R_1L + 2R_2L - \omega^3R_1L^2C)}{\omega^2(L^2 + 2R_1R_2LC) - R_2(R_1+R_2)}$$

$$\Rightarrow \varphi = \arctan\left(-\frac{R_2}{\omega L}\right) - \arctan\left[-\frac{\omega(R_1R_2^2C + R_1L + 2R_2L - \omega^3R_1L^2C}{\omega^2(L^2 + 2R_1R_2LC - R_2(R_1+R_2)}\right]$$

$\Rightarrow \varphi = 0°$ setzen!

$$\frac{R_2}{\omega L} = \frac{\omega(R_1R_2^2C + R_1L + 2R_2L - \omega^3R_1L^2C)}{\omega^2(L^2 + 2R_1R_2LC - R_2(R_1+R_2)}$$

$$\Rightarrow \quad \frac{R_2}{\omega L}[\omega^2(L^2 + 2R_1R_2LC) - R_2(R_1+R_2) = \omega(R_1R_2^2C + R_1L + 2R_2L) - \omega^3R_1L^2C$$

$$R_2\omega L + 2\omega R_1R_2^2C - \frac{R_2^2}{\omega L}(R_1+R_2) = \omega R_1R_2^2C + \omega R_1L + 2\omega R_2L - \omega^3R_1L^2C \quad / \cdot \omega L$$

[Teil 4]

Übertragungsfunktion Bandpass

$$R_2 \omega^2 L^2 + 2\omega^2 R_1 R_2^2 LC - R_2^2 (R_1+R_2) = \omega^2 R_1 R_2^2 LC + \omega^2 R_1 L + 2\omega^2 R_2 L^2 - \omega^4 R_1 L^3 C$$

$$\omega^4 R_1 L^3 C + 2\omega^2 R_1 R_2^2 LC - \omega^2 R_1 R_2^2 LC + R_2 \omega^2 L^2 - 2 R_2 \omega^2 L^2 - \omega^2 R_1 L^2 - R_2^2 (R_1+R_2) = 0$$

$$\Rightarrow \omega^4 R_1 L^3 C + \omega^2 R_1 R_2^2 LC - R_2 \omega^2 L^2 - \omega^2 R_1 L^2 - R_2^2 (R_1+R_2) = 0$$

$$\omega^4 + \omega^2 \frac{\cancel{R_1} R_2^2 \cancel{L} \cancel{C}}{\cancel{R_1} L^{\cancel{3}2} \cancel{C}} - \frac{R_2 \omega^2 \cancel{L^2}}{R_1 L^3 C} - \omega^2 \frac{\cancel{R_1} \cancel{L^2}}{\cancel{R_1} L^3 C} - \frac{R_2^2}{R_1 L^3 C} (R_1+R_2) = 0$$

$$\omega^4 + \left(\frac{R_2^2}{L^2} - \frac{R_2}{R_1} \cdot \frac{1}{LC} - \frac{1}{LC} \right) \omega^2 - \frac{R_2^2}{L^3 C} - \frac{R_2^3}{R_1 L^3 C} = 0$$

$\Rightarrow$ $\omega^2 = x$ setzen!

$$x^2 + \left(\frac{R_2}{L^2} - \frac{1}{LC} \left(\frac{R_2}{R_1} + 1 \right) \right) x - \frac{R_2^2}{L^3 C} \left(\frac{R_2}{R_1} + 1 \right) = 0$$

[Teil 5]

Übertragungsfunktion Bandpass

$$\Rightarrow x_{1/2} = \frac{-\left(\frac{R_2}{L^2} - \frac{1}{LC}\left(\frac{R_2}{R_1}+1\right)\right)}{2} \pm \sqrt{\left(\frac{\frac{R_2}{L^2} - \frac{1}{LC}\left(\frac{R_2}{R_1}+1\right)}{2}\right)^2 + \frac{R_2^{\,2}}{L^3 C}\cdot\left(\frac{R_2}{R_1}+1\right)}$$

$$\Rightarrow f_{res.} = \frac{\sqrt{x}}{2\pi} = \frac{\sqrt{\omega^2}}{2\pi} = \frac{\omega}{2\pi}$$